Part A

1. A
2. B
3. C
4. A
5. D
6. D
7. D
8. C
9. C
10. A
11. B
12. C
13. A
14. C
15. C
16. D
17. A
18. B
19. D
20. D
21. C
22. C
23. B
24. C
25. B
26. B
27. D
28. C
29. C
30. A

Part B

31. D
32. C
33. C
34. C
35. D
36. B
37. C
38. D
39. C
40. D
41. B
42. C
43. D
44. B
45. C

CALCULUS AB

SECTION I, Part A Solutions

1. Correct answer: (A)

Answer choice A is false. $\lim_{x \to 2} f(x)$ does not exist because the left- and right-hand limits are not equal.

$$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$$

2. Correct answer: (B)

Using the definition of a derivative, the derivative of the function $f(x)$ at $x = a$ can be found as $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$. In our case, it is the derivative of $f(x) = 25x^5$ at $x = \frac{1}{5}$.

$$f'(x) = 25 \cdot 5x^{5-1} = 125x^4$$

$$f'\left(\frac{1}{5}\right) = 125 \cdot \left(\frac{1}{5}\right)^4 = 125 \cdot \frac{1}{625} = \frac{1}{5}$$

3. Correct answer: (C)

$f(x) = k$ for $x = 4$, so $f(4) = k$.

To find the value of k where f is continuous at $x = 4$, we need to find $\lim_{x \to 4} f(x) = f(4)$.

$$f(4) = k = \lim_{x \to 4} \frac{\sqrt{3x+4} - \sqrt{2x+8}}{x-4} = \lim_{x \to 4} \frac{\sqrt{3x+4} - \sqrt{2x+8}}{x-4} \cdot \frac{\sqrt{3x+4} + \sqrt{2x+8}}{\sqrt{3x+4} + \sqrt{2x+8}}$$

$$= \lim_{x \to 4} \frac{(3x+4) - (2x+8)}{(x-4)(\sqrt{3x+4} + \sqrt{2x+8})} = \lim_{x \to 4} \frac{x-4}{(x-4)(\sqrt{3x+4} + \sqrt{2x+8})}$$

$$= \lim_{x \to 4} \frac{1}{\sqrt{3x+4} + \sqrt{2x+8}} = \frac{1}{\sqrt{16} + \sqrt{16}} = \frac{1}{4+4} = \frac{1}{8}$$

4. Correct answer: (A)

Substitution gives the indeterminate form $\frac{0}{0}$. Use L'Hospital's Rule.

$$\lim_{x \to 0} \frac{-2\sin 2x - 2\cos 2x}{1 + \cos x} = \frac{-2\sin 0 - 2\cos 0}{1 + \cos 0} = \frac{-2 \cdot 0 - 2 \cdot 1}{1+1} = \frac{-2}{2} = -1$$

5. Correct answer: (D)

Use the product rule:

$$\frac{d}{dx}(x\cos x) = \frac{d}{dx}(x) \cdot \cos x + x \cdot \frac{d}{dx}(\cos x) = 1 \cdot \cos x + x(-\sin x)$$

$$= \cos x - x\sin x$$

6. Correct answer: (D)

$$f'(x) = 2x + \frac{4}{x^2} + \frac{2}{\sqrt{2x-1}}$$

$$f'(1) = 2(1) + \frac{4}{1^2} + \frac{2}{\sqrt{2(1)-1}} = 2 + 4 + 2 = 8$$

7. Correct answer: (D)

Use the chain rule.

$$y' = 4(\sqrt{x} + \sin x)^3 \cdot \frac{d}{dx}(\sqrt{x} + \sin x) = 4(\sqrt{x} + \sin x)^3 \left(\frac{1}{2\sqrt{x}} + \cos x \right)$$

8. Correct answer: (C)

Use the quotient rule.

$$f'(x) = \frac{\frac{d}{dx}(x) \cdot \tan x - x \cdot \frac{d}{dx}(\tan x)}{(\tan x)^2} = \frac{1 \cdot \tan x - x \sec^2 x}{\tan^2 x} = \frac{\tan x - x \sec^2 x}{\tan^2 x}$$

$$= \frac{1}{\tan x} - \frac{x \sec^2 x}{\tan^2 x} = \cot x - x \cdot \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \cot x - \frac{x}{\sin^2 x} = \cot x - x \csc^2 x$$

Then

$$f'\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}} - \frac{\pi}{3}\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{1}{\sqrt{3}} - \frac{4\pi}{9} = \frac{3\sqrt{3} - 4\pi}{9}$$

9. Correct answer: (C)

The stem of the question means $f'(3) = 9$. So f is differentiable at $x = 3$ and therefore continuous at $x = 3$. We know nothing about the continuity of f'. Therefore, the only correct statements are I and II.

10. Correct answer: (A)

The average value of the function $f(x)$ on the interval $[a, b]$ is

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

$$f_{ave} = \frac{1}{2-0}\int_0^2 x^2\sqrt{3x^3+1}\ dx$$

Let $u = 3x^3 + 1$, $du = 9x^2\ dx$, and $\frac{du}{9} = x^2\ dx$.

$$f_{ave} = \frac{1}{2}\int_0^2 \frac{1}{9}\sqrt{u}\ du = \frac{1}{9\cdot 2}\int_0^2 \sqrt{u}\ du = \frac{1}{18}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_0^2 = \frac{1}{27}(3x^3+1)^{\frac{3}{2}}\Big|_0^2$$

$$= \frac{1}{27}((3(2)^3+1)^{\frac{3}{2}} - (3(0)^3+1)^{\frac{3}{2}}) = \frac{1}{27}((25)^{\frac{3}{2}} - (1)^{\frac{3}{2}}) = \frac{1}{27}(125-1) = \frac{124}{27}$$

11. Correct answer: (B)

To find when a function is decreasing, you must first take the derivative, then set it equal to 0, and then find between which zero values the function is negative.

$$f'(x) = 3x^2 - 75$$

$$3x^2 - 75 = 0$$

$$3(x^2 - 25) = 0$$

$$(x - 5)(x + 5) = 0$$

$$x = \pm 5$$

Now test values on all sides of these to find when the function is negative, and therefore decreasing. You could pick -6, 0, and 6.

$$f'(-6) = 3(-6)^2 - 75 = 3 \cdot 36 - 75 = 108 - 75 = 33$$

$$f'(0) = 3(0) - 75 = -75$$

$$f'(6) = 3(6)^2 - 75 = 3 \cdot 36 - 75 = 108 - 75 = 33$$

Since the values that are negative occur when $x = 0$, the function is decreasing on the intervals that include these values, so the function is decreasing on $[-5,5]$.

12. Correct answer: (C)

Using one of the integration formulas, $\int \csc^2 x \ dx = -\cot x + C$.

13. Correct answer: (A)

The particle changes direction when velocity is 0, so completing the square on $9t^2 - 2t - 7 = 0$ gives $t = -\frac{7}{9}, 1$, but $t = 1$ is the only solution on the given interval $t = [0,2]$.

Then total distance traveled by the particle from time $t = 0$ to $t = 2$ is

$$\int_0^2 |9t^2 - 2t - 7| \ dt = \left| \int_0^1 9t^2 - 2t - 7 \ dt \right| + \left| \int_1^2 |9t^2 - 2t - 7| \ dt \right|$$

$$= \left| 3t^3 - t^2 - 7t \Big|_0^1 \right| + \left| 3t^3 - t^2 - 7t \Big|_1^2 \right| = |3 - 1 - 7| + |24 - 4 - 14 - (3 - 1 - 7)|$$

$$= |-5| + |11| = 5 + 11 = 16$$

14. Correct answer: (C)

Right-hand sum: $f(t_1)\Delta t + f(t_2)\Delta t + f(t_3)\Delta t + f(t_4)\Delta t$.

Substitute to get

$$5.0(8 - 5) + 4.0(11 - 8) + 3.5(15 - 11) + 2.0(19 - 15)$$

$$= 5(3) + 4(3) + 3.5(4) + 2(4) = 15 + 12 + 14 + 8 = 49$$

The approximation of the number of liters of water that are in the tank at time $t = 19$ hours is $30 + 49 = 79$ liters.

15. Correct answer: (C)

Since both of the equations are functions, these are upper and lower curves. Find the points of intersection by setting the curves equal to each other.

$$3x^2 + x - 2 = x + 1$$

$$3x^2 - 3 = 0$$

$$x^2 - 1 = 0$$

$$(x+1)(x-1)=0$$

$$x=-1,\ 1$$

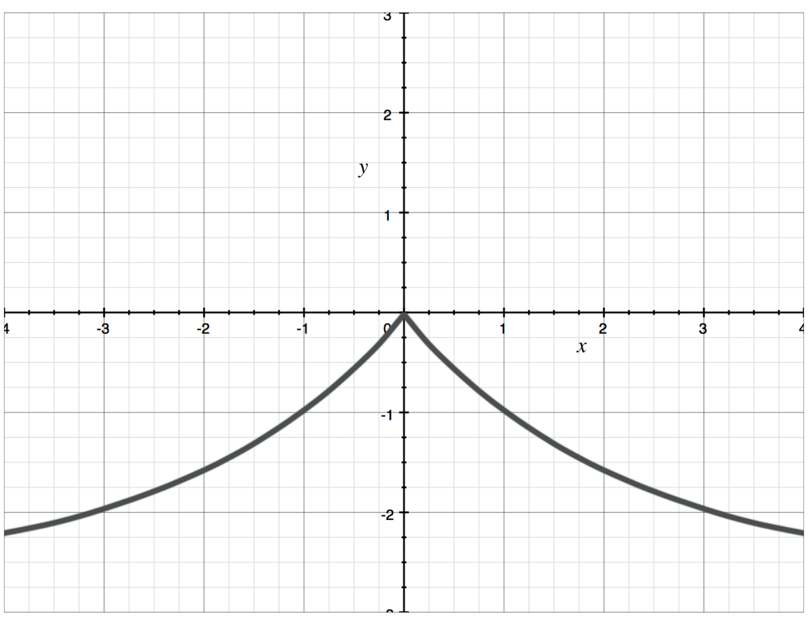
The function $y=x+1$ is the upper curve, so the integral to find the area between the two curves is

$$\int_{-1}^{1}(x+1)-(3x^2+x-2)\ dx=\int_{-1}^{1}x+1-3x^2-x+2\ dx$$

$$\int_{-1}^{1}-3x^2+3\ dx=-\frac{3}{3}x^3+3x\Big|_{-1}^{1}=-(1)^3+3(1)-(-(-1)^3+3(-1))$$

$$-1+3-(1-3)=2-(-2)=4$$

16. Correct answer: (D)



f is continuous at $x=0$.

$$f'(x)=-\frac{1}{2\sqrt{|x|}}$$

Therefore, f is not differentiable at $x=0$.

17. Correct answer: (A)

Solve using substitution. Set $u = 3x + 4$ and $\frac{du}{dx} = 3$, so $du = 3\ dx$ and $dx = \frac{du}{3}$. The new limits of integration, in terms of u, will be $u = [3(0) + 4, 3(2) + 4] = [4,10]$.

$$\int_0^2 3(3x+4)^4\ dx = \int_4^{10} 3u^4\ \frac{du}{3} = \int_4^{10} u^4\ du$$

18. Correct answer: (B)

$$\int_0^3 f(x)\ dx = \int_0^1 f(x)\ dx + \int_1^3 f(x)\ dx$$

$$= \int_0^1 (2x+4)\ dx + \int_1^3 (-3x+9)\ dx$$

$$= x^2 + 4x\Big|_0^1 + \left[-\frac{3x^2}{2} + 9x\right]_1^3$$

$$= (1 + 4 - 0) + \left(-\frac{27}{2} + 27 - \left(-\frac{3}{2} + 9\right)\right)$$

$$= 5 + \frac{27}{2} - \frac{15}{2} = 5 + 6 = 11$$

19. Correct answer: (D)

First, find $f(g(x))$. Substitute $g(x)$ for x into $f(x)$.

$$f(g(x)) = 2(2x+3)^3$$

Then

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}f(g(x)) = 2 \cdot 3(2x+3)^2 \cdot 2 = 12(2x+3)^2$$

Now evaluate the derivative at $x = 1$.

$$\frac{d}{dx}(f(g(1))) = 12(2(1)+3)^2 = 12(5)^2 = 12 \cdot 25 = 300$$

20. Correct answer: (D)

From the graph we see that $g(5) = \int_0^5 f(t)\ dt$ is positive.

$$g'(x) = \frac{d}{dx}\int_0^x f(t)\ dt = f(x)$$

Therefore

$$g'(5) = f(5) = 0$$

$$g'(x) = f(x)$$

$$g''(x) = f'(x)$$

So $g''(5) = f'(5)$, which is the slope at $x = 5$. From the graph we see that the slope at $x = 5$ is negative. Therefore, $g''(5) < g'(5) < g(5)$.

21. Correct answer: (C)

At rest, $v(t) = 0$. Also, $v(t) = x'(t)$.

Use the product rule.

$$v(t) = 2te^{-3t} - 3t^2e^{-3t}$$

Now solve $v(t) = 0$.

$$2te^{-3t} - 3t^2e^{-3t} = 0$$

$$e^{-3t}(2t - 3t^2) = 0$$

$$e^{-3t}(t)(2 - 3t) = 0$$

$$t = 0, \frac{2}{3}$$

22. Correct answer: (C)

$$\frac{d}{dx}\int_0^x g(t)\ dt = g(x)$$

Therefore

$$\frac{d}{dx}(f(x)) = f'(x) = g(x)$$

The graph shows that f is decreasing on the interval $(-4,0)$ and increasing on the interval $(0,4)$. This means the graph of the derivative of f is negative on the interval $(-4,0)$ and positive on the interval $(0,4)$.

23. Correct answer: (B)

The slope of the tangent line is 0, which means that $f'(x)=0$.

Find the derivative using the quotient rule.

$$f'(x)=\frac{2x(2x+1)-x^2(2)}{(2x+1)^2}=\frac{4x^2+2x-2x^2}{(2x+1)^2}=\frac{2x^2+2x}{(2x+1)^2}$$

Solve the equation.

$$\frac{2x^2+2x}{(2x+1)^2}=0$$

$$2x^2+2x=0$$

$$x(x+1)=0$$

$$x=-1,\ 0$$

Then $f(-1)=\frac{(-1)^2}{2(-1)+1}=-1$ and $f(0)=\frac{0^2}{2(0)+1}=0$.

24. Correct answer: (C)

Since g is the inverse function of f, if $f(1)=0$, then $g(0)=1$.

Remember that if $f^{-1}(x)=g(x)$, then $g'(x)=\frac{1}{f'(g(x))}$. Therefore,

$$g'(0)=\frac{1}{f'(g(0))}=\frac{1}{f'(1)}$$

Now find the derivative of $f(x)$ at $x=1$.

$$f'(x)=5x^4$$

$$f'(1) = 5 \cdot (1)^4 = 5$$

So $g'(0) = \frac{1}{5}$.

25. Correct answer: (B)

To find the vertical asymptotes, set the denominator equal to 0. Therefore, we have

$$c - x^2 = 0$$

$$c - 2^2 = 0$$

$$c - 4 = 0$$

$$c = 4$$

For $x = -2$ we also get $c = 4$.

If the numerator and denominator are of equal degree, then the ratio of coefficients will give the horizontal asymptote. In our case, we have ax^2 in the numerator and $-x^2$ in the denominator. Therefore, $\frac{ax^2}{-x^2} = -a = 5$, so $a = -5$. Now $a + c = -5 + 4 = -1$.

26. Correct answer: (B)

Start by finding the derivative of the function.

$$f'(x) = e^{2x} + 2xe^{2x}$$

At critical points, $f'(x) = 0$.

$$e^{2x} + 2xe^{2x} = 0$$

$$e^{2x}(1 + 2x) = 0$$

$$1 + 2x = 0$$

$$x = -\frac{1}{2}$$

$$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)e^{2\left(-\frac{1}{2}\right)} = -\frac{1}{2e}$$

The absolute minimum value is $-\frac{1}{2e}$.

27. Correct answer: (D)

Separate variables.

$$\frac{dy}{dt} = k$$

$$dy = k\ dt$$

Integrate both sides.

$$y = kt + c$$

Use the values in the table.

$$3 = k(0) + c$$

$$c = 3$$

$$7 = k(1) + 3$$

$k = 4$

Therefore, we get $y = 4t + 3$.

28. Correct answer: (C)

Start by finding derivative of the function.

$$f'(x) = 2xe^{-kx} + (x^2 + 1)(-ke^{-kx}) = e^{-kx}(2x - kx^2 - k)$$

At critical point $f'(x) = 0$, DNE, we get:

$$e^{-kx}(2x - kx^2 - k) = 0$$

$e^{-kx} \neq 0$, so $2x - kx^2 - k = 0$

We know that critical point is at $x = 1$. Substitute and find the value of k.

$$2(1) - k(1)^2 - k = 0$$

$$2 - k - k = 0$$

$$2 = 2k$$

$$k = 1$$

29. Correct answer: (C)

First separate the variables. Multiply both sides by dx and divide both sides by e^{-y}.

$$\frac{dy}{dx} = xe^{-y}$$

$$\frac{dy}{e^{-y}} = x\ dx$$

$$e^y\ dy = x\ dx$$

Integrate both sides.

$$\int e^y\ dy = \int x\ dx$$

$$e^y + c_1 = \frac{x^2}{2} + c_2$$

$e^y = \dfrac{x^2}{2} + c$, where $c = c_2 - c_1$

$$\ln e^y = \ln\left(\frac{x^2}{2} + c\right)$$

$$y = \ln\left(\frac{x^2}{2} + c\right)$$

Since $y(0) = 1$, substitute and find c.

$$0 = \ln\left(\frac{1}{2} + c\right)$$

We remember that $\ln 1 = 0$, so $\dfrac{1}{2} + c = 1$, and $c = \dfrac{1}{2}$.

Therefore, we get $y = \ln\left(\dfrac{x^2}{2} + \dfrac{1}{2}\right)$.

30. Correct answer: (A)

Since the position is $x(t)=\cos t+\sin t$, the velocity is $v(t)=x'(t)=-\sin t+\cos t$.

We first need to find the point where velocity is first equal to 0.

$$-\sin t+\cos t=0$$

$\cos t=\sin t$, which is true when $t=\frac{\pi}{4}$.

Now we know that

$$a(t)=v'(t)=-\cos t-\sin t$$

and

$$a\left(\frac{\pi}{4}\right)=-\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}\right)$$

$$=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}=-\frac{2\sqrt{2}}{2}=-\sqrt{2}$$

CALCULUS AB

SECTION I, Part B Solutions

31. Correct answer: (D)

We can rewrite the equation as $\frac{dy}{dx}=\frac{2y-2xy^3}{3x^2y^2+2x}$.

Now find $\frac{dy}{dx}$ at the point $(-1,1)$.

$$\frac{dy}{dx}(-1,1)=\frac{2(1)-2(-1)(1)^3}{3(-1)^2(1)^2+2(-1)}=\frac{2+2}{3-2}=\frac{4}{1}=4$$

Then use the quotient rule.

$$\frac{d^2y}{dx^2}=\frac{(2y-2xy^3)'(3x^2y^2+2x)-(3x^2y^2+2x)'(2y-2xy^3)}{(3x^2y^2+2x)^2}$$

$$=\frac{\left(2\frac{dy}{dx}-2y^3-6xy^2\frac{dy}{dx}\right)(3x^2y^2+2x)-\left(6xy^2+6x^2y\frac{dy}{dx}+2\right)(2y-2xy^3)}{(3x^2y^2+2x)^2}$$

$$=\frac{\left(2(4)-2(1)-6(-1)(1)(4)\right)\left(3(1)(1)+2(-1)\right)-\left(6(-1)(1)+6(1)(1)(4)+2\right)\left(2(1)-2(-1)(1)\right)}{\left(3(1)(1)+2(-1)\right)^2}$$

$$=\frac{(8-2+24)(1)-(20)(0)}{1}=30$$

32. Correct answer: (C)

To find the slope of the line tangent to the graph, we need to find $f'(x)$.

$$f'(x) = 30xe^{3x^2}$$

We want to have $30xe^{3x^2} = 1$.

Graph the derivative function and the function $y = 1$, then find the intersection to get $x = 0.033$.

33. Correct answer: (C)

The Intermediate Value Theorem states that if f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ differ in sign, the theorem guarantees the existence of at least one zero of f in the closed interval $[a, b]$.

34. Correct answer: (C)

Since the graph crosses the x-axis at one point, then the y-value is equal to 0. Substitute $y = 0$ into the equation and find the x-value.

$$0 = 3\ln(\sec x)$$

$\ln 1 = 0$, so $\sec x = 1$ and $x = 2\pi n$, where n is any integer

We have the interval $[6,7]$, so $x = 2\pi \approx 6.2832$.

To find the slope, we need to find y'.

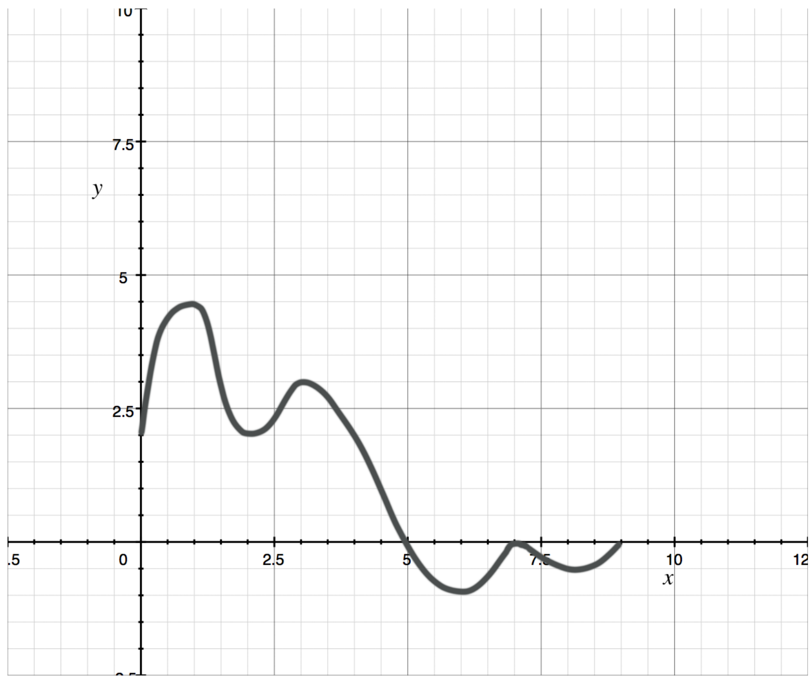
$$y' = 3\frac{\sec x \tan x}{\sec x} = 3\tan x$$

$$y'(2\pi) = y'(6.2832) = 3\tan 2\pi = 0$$

35. Correct answer: (D)

Points of inflection occur where f' changes from increasing to decreasing or from decreasing to increasing. There are six such points. f' changes from positive to negative at $x = 5$, therefore f has a relative maximum at $x = 5$.

Since f' is decreasing on the interval $[3,6]$, the function f is concave down on the interval $[3,6]$. Therefore, only statements I and III are true.



36. Correct answer: (B)

$$f'(x) = 20x^4 - 120x^2$$

$$f''(x) = 80x^3 - 240x = 80x(x^2 - 3)$$

The graph of f is concave up where $f'' > 0$. $f'' > 0$ for $-\sqrt{3} < x < 0$ and $x > \sqrt{3}$.

37. Correct answer: (C)

Since $F'(x) = f(x)$, then F is an antiderivative of f. Therefore,

$$\int_2^5 f(2x)\ dx = \frac{1}{2}F(2x)\Big|_2^5 = \frac{1}{2}(F(10) - F(4))$$

38. Correct answer: (D)

The graph of f is concave down where $f'' < 0$. On the graph of f'', we see that $f'' < 0$ on $-2 < x < 2$.

39. Correct answer: (C)

The area of a triangle is given by the formula $A = \frac{1}{2}bh$. Take the derivative with respect to t of both sides of the equation.

$$\frac{dA}{dt} = \frac{1}{2}\left(h\frac{db}{dt} + b\frac{dh}{dt}\right)$$

Substitute the given rates

$$\frac{dA}{dt} = \frac{1}{2}(2h + 2b) = h + b$$

The area will be decreasing when $\frac{dA}{dt} < 0$, which is true when $h + b < 0$, or $h < -b$.

40. Correct answer: (D)

We know that $a(t) = t^2 + \cos t$ and $v(0) = 1$.

$$v(t) = \int a(t)\ dt$$

$$v(t) = \int t^2 + \cos t\ dt = \frac{t^3}{3} + \sin t + C$$

Substitute $v(0) = 1$ and solve for C.

$$0 + \sin 0 + C = -1$$

$$c = -1$$

Therefore

$$v(t) = \frac{t^3}{3} + \sin t - 1$$

Then we need to find t when $v(t) = 0$.

$$\frac{t^3}{3} + \sin t - 1 = 0$$

$$t \approx 0.881$$

41. Correct answer: (B)

Change the integration from x to t.

$t = 3x$, then $dt = 3\ dx$, or $dx = \frac{1}{3}\ dt$.

Use $t = 3x$ to find the new limits of integration.

If $x = 5$, then $t = 3 \cdot 5 = 15$

If $x = 11$, then $t = 3 \cdot 11 = 33$

Then we get

$$\int_{5}^{11} f(3x)\ dx = \frac{1}{3}\int_{15}^{33} f(t)\ dt$$

$$\frac{1}{3}\int_{15}^{33} f(t)\ dt = 15$$

$$\int_{15}^{33} f(t)\ dt = 15 \cdot 3 = 45$$

42. Correct answer: (C)

Points of inflection occur where f' changes from increasing to decreasing or from decreasing to increasing. According to the table, there are two inflection points.

43. Correct answer: (D)

Since both of the equations are functions, these are upper and lower curves. Find the points of intersection by setting the curves equal to each other.

$$3x^2 + x - 2 = x + 3$$

$$3x^2 - 5 = 0$$

$$x \approx \pm\sqrt{\frac{5}{3}}$$

The function $y = x + 3$ is the upper curve, so the integral to find the area between the two curves is

$$\int_{-\sqrt{\frac{5}{3}}}^{\sqrt{\frac{5}{3}}} (x+3)-(3x^2+x-2)\ dx = \int_{-\sqrt{\frac{5}{3}}}^{\sqrt{\frac{5}{3}}} x+3-3x^2-x+2\ dx$$

$$\int_{-\sqrt{\frac{5}{3}}}^{\sqrt{\frac{5}{3}}} 5-3x^2\ dx = 5x-x^3\Big|_{-\sqrt{\frac{5}{3}}}^{\sqrt{\frac{5}{3}}}$$

$$5\sqrt{\frac{5}{3}}-\left(\sqrt{\frac{5}{3}}\right)^3-\left[5\left(-\sqrt{\frac{5}{3}}\right)-\left(-\sqrt{\frac{5}{3}}\right)^3\right]$$

$$5\sqrt{\frac{5}{3}}-\left(\sqrt{\frac{5}{3}}\right)^3+5\sqrt{\frac{5}{3}}-\left(\sqrt{\frac{5}{3}}\right)^3=\frac{20\sqrt{5}}{3\sqrt{3}}\approx 8.607$$

44. Correct answer: (B)

Find the intersection points to find the interval $[a, b]$.

$$x^2+3=x+3$$

$$x^2-x+3-3=x-x+3-3$$

$$x^2-x=0$$

$$x(x-1)=0$$

$$x=0,\ 1$$

The interval $[a, b]$ is $[0,1]$, $f(x)=x+3$ because the line is above the parabola in the interval $[0,1]$, and $g(x)=x^2+3$.

$$V=\pi\int_a^b [f(x)]^2-[g(x)]^2\ dx=\pi\int_0^1 (x+3)^2-(x^2+3)^2\ dx$$

$$=\pi\int_0^1 x^2+6x+9-(x^4+6x^2+9)\ dx=\pi\int_0^1 -x^4-5x^2+6x\ dx$$

$$=\pi\left(-\frac{x^5}{5}-\frac{5}{3}x^3+3x^2\right)\Bigg|_0^1=\pi\left[-\frac{1^5}{5}-\frac{5}{3}(1)^3+3(1)^2-\left(-\frac{0^5}{5}-\frac{5}{3}(0)^3+3(0)^2\right)\right]$$

$$=\pi\left(-\frac{1}{5}-\frac{5}{3}+3-0\right)=\pi\left(-\frac{3}{15}-\frac{25}{15}+\frac{45}{15}\right)=\frac{17}{15}\pi\approx 3.560$$

45. Correct answer: (C)

Let x be the distance from the train to the intersection. Then $\frac{dx}{dt}=65$.

Use the Pythagorean theorem.

$$S^2=x^2+90^2$$

Find the derivative.

$$2S\frac{dS}{dt}=2x\frac{dx}{dt},\text{ or }\frac{dS}{dt}=\frac{x}{S}\frac{dx}{dt}$$

After 8 seconds, $x=65\cdot 8=520$ and so $S=\sqrt{90^2+520^2}\approx 527.731$. Therefore,

$$\frac{dS}{dt}=\frac{520}{527.731}\cdot(65)\approx 64.048$$

CALCULUS AB

SECTION II, Part A Solutions

1. Solution:

a. To find how many men enter the cableway during the time interval $0 \leq t \leq 150$, we need to find the integral of $f(t)$ with the limits of integration $0 \leq t \leq 150$.

$$\int_0^{150} \frac{7}{1{,}250}t^2\left(\frac{150-t}{150}\right)^6\ dt = 75$$

b. Let $M(t)$ be the number of men in line. We know that there are 5 men in line at time $t = 0$. Therefore, $M(0) = 5$.

$$M(150) - M(0) = \int_0^{150} f(t) - 0.4\ dt$$

$$M(150) = 5 + \int_0^{150} f(t) - 0.4\ dt = \int_0^{150} \frac{7}{1{,}250}t^2\left(\frac{150-t}{150}\right)^6 - 0.4\ dt = 20$$

c. Let P be the first time there are no men in line.

$$M(P) - M(150) = \int_{150}^{k} -0.4\ dt$$

$$0 - 20 = -0.4(k - 150)$$

$k = 200$ minutes

d. $\dfrac{dM}{dt} = f(t) - 0.4$

Find the time when the number of men in the line for the cableway is at a minimum.

$$f(t) - 0.4 = 0$$

$$\frac{7}{1{,}250}t^2\left(\frac{150 - t}{150}\right)^6 - 0.4 = 0$$

$$t_1 \approx 10.5093$$

$$t_2 \approx 78.7026$$

Then

$$M(10.5093) - M(0) = \int_0^{10.5093} f(t) - 0.4\ dt$$

$$M(10.5093) = 5 + \int_0^{10.5093} f(t) - 0.4\ dt$$

$$M(10.5093) \approx 2.3684$$

For $0 \le t \le 150$, the minimum number of men in line is 2 men at $t = 10.5093$ by the extreme value theorem.

2. Solution:

a. We remember that $a(t) = v'(t)$, so the first thing is to find the derivative of the velocity at $t = 3$. Using a calculator, $v'(3) = 3.009$.

Now find $v(3)$. Using a calculator, $v(3) \approx -1.125 < 0$. Speed is decreasing since $a(3) > 0$ and $v(3) < 0$.

b. $v(t) = 0$ when $t \approx 3.316$. $v(t)$ changes from positive to negative at $t = 3.316$. Therefore, the particle changes direction at this time.

c. Total distance $= \int_0^4 |v(t)| \ dt = 7.170$.

d. $x(3.316) = x(0) + \int_0^{3.316} v(t) \ dt = 2 + \int_0^{3.316} v(t) \ dt = -4.086$

Since the total distance from $t = 0$ to $t = 4$ is 7.170, the particle is still to the left of the origin at $t = 4$. So the greatest distance from the origin is 4.086.

CALCULUS AB

SECTION II, Part B Solutions

3. Solution:

a. $f'(x) = g(x)$

$$f(x) = \int g(x)\ dx$$

$$f(7) - f(-1) = \int_{-1}^{7} g(x)\ dx$$

$$f(7) = f(-1) + \int_{-1}^{7} g(x)\ dx$$

$$f(7) = 3 + \left(1(3) + \frac{1}{2}(1)(2) + \frac{1}{2}(5)(5) - \frac{1}{2}(1)(2)\right)$$

$$= 3 + \left(3 + \frac{25}{2}\right) = 3 + \frac{31}{2} = \frac{37}{2}$$

b. $\int_{-6}^{-1} g(x)\ dx = \int_{-6}^{-3} g(x)\ dx + \int_{-3}^{-1} g(x)\ dx$

$$= \int_{-6}^{-3} -2(x+4)^2 + 5\ dx + \int_{-3}^{-1} 3\ dx$$

$$= \left[-\frac{2}{3}(x+4)^3 + 5x\right]_{-6}^{-3} + 3x\Big|_{-3}^{-1}$$

$$= -\frac{2}{3}\left[(-3+4)^3 - (-6+4)^3\right] + 5(-3-(-6)) + 3(-1-(-3))$$

$$= -\frac{2}{3}(1+8) + 5(3) + 3(2)$$

$$= -6 + 15 + 6 = 15$$

c. f is decreasing when $f' < 0$ or $g(x) < 0$ and f is concave down when $f'' > 0$ or when the slope of $g(x) < 0$. The graph of f is decreasing and concave down on $6 < x < 7$ because $g(x) < 0$ and $g(x)$ is decreasing on this interval.

d. Points of inflection occur where f' changes from increasing to decreasing, or from decreasing to increasing. There are two inflection points at $x = -4$ and $x = 0$.

4. Solution

a. $T'(9) = \dfrac{T(10) - T(8)}{10 - 8}$

$$= \frac{51 - 60}{10 - 8} = \frac{-9}{2} = -4.5\ C^\circ\text{/cm}$$

b. The average temperature of the wire is $\dfrac{1}{10}\displaystyle\int_0^{10} T(x)\ dx$. Let $A = \displaystyle\int_0^{10} T(x)\ dx$, then the trapezoidal approximation for A is

$$\frac{100+90}{2} \cdot 2 + \frac{90+81}{2} \cdot 1 + \frac{81+73}{2} \cdot 2 + \frac{73+60}{2} \cdot 3 + \frac{60+51}{2} \cdot 2 = 740$$

The average temperature is $\dfrac{1}{10}A = \dfrac{1}{10} \cdot 740 = 74\ C^\circ$.

c. $\displaystyle\int_0^{10} T'(x)\ dx = T(10) - T(0) = 51 - 100 = -49\ C^\circ$

The temperature drops $49\ C^\circ$ from the heated end of the wire to the other end of the wire.

d. $\dfrac{T(5) - T(3)}{5 - 3} = \dfrac{73 - 81}{2} = \dfrac{-8}{2} = -4$

Because T is differentiable on $3 \le x \le 5$, T is continuous on $3 \le x \le 5$. By the Mean Value Theorem, there exists a value c, $3 < c < 5$, such that $T'(c) = -4$.

5. Solution:

a. The average rate of change of f on the interval $\dfrac{\pi}{2} < x < 2\pi$ is

$$\frac{f(2\pi) - f\left(\frac{\pi}{2}\right)}{2\pi - \frac{\pi}{2}} = \frac{-e^{\frac{3\pi}{2}}}{\frac{3\pi}{2}} = \frac{-2e^{\frac{3\pi}{2}}}{3\pi}$$

because

$$f(2\pi) = e^{3(2\pi)}\sin 6\pi = e^{6\pi}(0) = 0$$

$$f\left(\frac{\pi}{2}\right) = e^{3\left(\frac{\pi}{2}\right)}\sin\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}}(-1) = -e^{\frac{3\pi}{2}}$$

b. To find the slope of the line tangent to the graph of f at $x = \pi$, we need to find $f'(\pi)$.

$$f'(x) = 3e^{3x}\sin 3x + 3e^{3x}\cos 3x$$

$$f'(\pi) = 3e^{3\pi}\sin 3\pi + 3e^{3\pi}\cos 3\pi$$

$$= 0 + 3e^{3\pi}(-1) = -3e^{3\pi}$$

Therefore, the slope of the line tangent to the graph of f at $x = \pi$ is $-3e^{3\pi}$.

c. $f'(x) = 0$

From part b, we know that $f'(x) = 3e^{3x}\sin 3x + 3e^{3x}\cos 3x$. Therefore,

$$3e^{3x}\sin 3x + 3e^{3x}\cos 3x = 0$$

$$3e^{3x}(\sin 3x + \cos 3x) = 0$$

$$3e^{3x} \neq 0$$

$$\sin 3x + \cos 3x = 0$$

$$\sin 3x = -\cos 3x$$

$$3x = \frac{3\pi}{4}$$

$$x = \frac{\pi}{4}$$

To find the absolute maximum value of f on $0 \leq x \leq \pi$, we need to evaluate $f(x)$ at $x = 0$, $x = \frac{\pi}{4}$, and $x = \pi$.

$$f(0) = e^0 \sin 0 = 0$$

$$f\left(\frac{\pi}{4}\right) = e^{\frac{3\pi}{4}} \sin\left(\frac{3\pi}{4}\right) = e^{\frac{3\pi}{4}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}$$

$$f(\pi) = e^{3\pi}\sin 3\pi = 0$$

The absolute maximum value of f on $0 \leq x \leq \pi$ is $\frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}$.

d. $\lim_{x\to\pi} \frac{g(x)}{f(x)} = \frac{\lim_{x\to\pi} g(x)}{\lim_{x\to\pi} f(x)}$

Because g is differentiable, g is continuous.

$$\lim_{x\to\pi} g(x) = g(\pi) = 0$$

$$\lim_{x\to\pi} f(x) = \lim_{x\to\pi} e^{3x}\sin 3x = e^{3\pi}(0) = 0$$

By L'Hospital's Rule,

$$\lim_{x\to\pi} \frac{g(x)}{f(x)} = \lim_{x\to\pi} \frac{g'(x)}{f'(x)} = \frac{\lim_{x\to\pi} g'(x)}{\lim_{x\to\pi} f'(x)}$$

From the graph of g', we see that $\lim_{x\to\pi} g'(x) = -1$.

$$\lim_{x\to\pi} f'(x) = \lim_{x\to\pi} 3e^{3x}\sin 3x + 3e^{3x}\cos 3x$$

$$= 3e^{3\pi}\sin 3\pi + 3e^{3\pi}\cos 3\pi$$

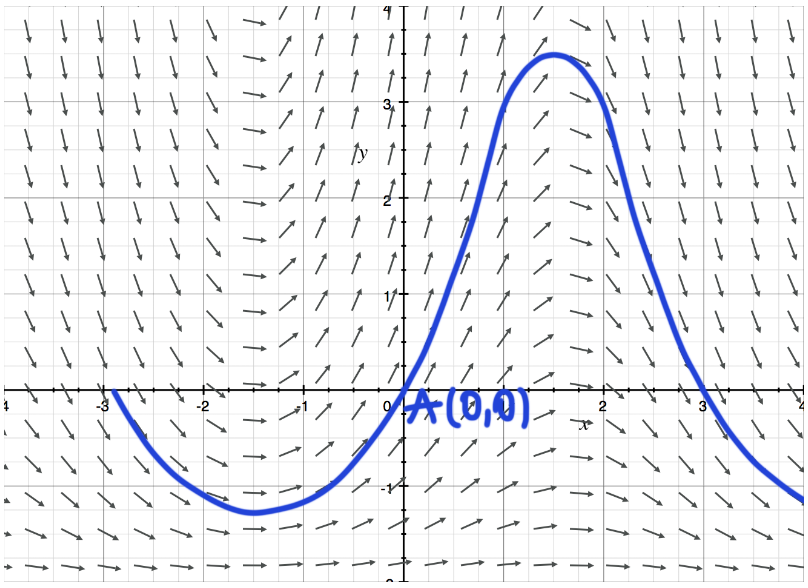
$$= 0 + 3e^{3\pi}(-1) = -3e^{3\pi}$$

Therefore

$$\lim_{x\to\pi} \frac{g(x)}{f(x)} = \frac{-1}{-3e^{3\pi}} = \frac{1}{3e^{3\pi}}$$

6. Solution:

a. First, we find the point $(0,0)$ on the graph function, and then we draw the curve using the function's slope. The resulting graph is the following:



b. First, we need to evaluate the slope at $(0,0)$.

$$\frac{dy}{dx}(0,0) = (0+2)\cos 0 = 2(1) = 2$$

An equation for the tangent line is $y - y_1 = m(x - x_1)$

Substitute and we get:

$$y - 0 = 2(x - 0)$$

$$y = 2x$$

Therefore

$$f(0.5) = 2(0.5) = 1$$

c. First separate the variables

$$\frac{dy}{dx} = (y+2)\cos x$$

$$\frac{dy}{y+2} = \cos x \ dx$$

Integrate both sides.

$$\int \frac{dy}{y+2} = \int \cos x \ dx$$

$$\ln|y+2| + c_1 = \sin x + c_2$$

$\ln|y+2| = \sin x + c$, where $c = c_2 - c_1$

Substitute the initial condition $f(0) = 0$ and solve for c.

$$\ln|0+2| = \sin 0 + c$$

$$\ln 2 = c$$

$$\ln|y+2| = \sin x + \ln 2$$

Because $y(0) = 0$, $y > -2$, so $|y+2| = y+2$.

$$\ln(y+2) = \sin x + \ln 2$$

$$y + 2 = e^{\sin x + \ln 2}$$

$$y + 2 = 2e^{\sin x}$$

$$y = 2e^{\sin x} - 2$$